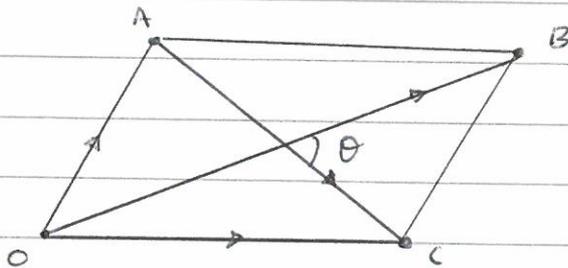


$$1) \quad x + \frac{1}{1-x} + \frac{2}{1+x}$$

$$= \frac{x(1-x)(1+x) + 1(1+x) + 2(1-x)}{(1-x)(1+x)}$$

$$= \frac{x - x^3 + 1 + x + 2 - 2x}{(1-x)(1+x)} = \frac{3 - x^3}{(1-x)(1+x)}$$

2)



$$\vec{AC} = -\begin{pmatrix} 2 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -11 \\ 4 \end{pmatrix}$$

$$\vec{OB} = \begin{pmatrix} 5 \\ 5 \\ 8 \end{pmatrix}$$

$$\cos \theta = \frac{\vec{AC} \cdot \vec{OB}}{|\vec{AC}| |\vec{OB}|} = \frac{1 \times 5 - 5 \times 11 + 8 \times 4}{\sqrt{1+121+16} \sqrt{25+25+64}} = -0.1435$$

$$\theta = 98.3^\circ \text{ (obtuse angle)}$$

$$\therefore \text{Acute angle} = 180 - 98.3 = 81.7^\circ$$

$$3 \quad (i) \quad (1-2x)^{-\frac{1}{2}}$$

$$= 1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2}$$

$$= 1 + x + \frac{3x^2}{2}$$

$$(ii) \quad \frac{x+3}{\sqrt{1-2x}} = (x+3)(1-2x)^{-\frac{1}{2}}$$

$$= (x+3)\left(1+x+\frac{3x^2}{2}\right)$$

Coefficient of $x^2 = 1 + \frac{9}{2} = \frac{11}{2}$

$$4. \quad \int_0^{\frac{\pi}{4}} \frac{1-2\sin^2 x}{1+2\sin x \cos x} dx = \int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1+\sin 2x} dx$$

$$= \left[\frac{1}{2} \ln(1+\sin 2x) \right]_0^{\frac{\pi}{4}}$$

Note: this is the type where the denominator differentiates to give a multiple of the numerator. Hence logs.

$$= \frac{1}{2} \ln\left(1+\sin \frac{\pi}{2}\right) - \frac{1}{2} \ln(1+\sin 0)$$

$$= \frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 = \frac{1}{2} \ln 2$$

5. (i) A and B are not parallel since direction vectors are not multiples of each other.

Assume A and B intersect

$$\textcircled{1} \quad 1 - s = 2 + t$$

$$\textcircled{2} \quad 4 + 2s = 8 + 3t$$

$$\textcircled{3} \quad 1 + 2s = 2 + 5t$$

$$\textcircled{2} - \textcircled{3} \quad 3 = 6 - 2t \quad \Rightarrow \quad t = \frac{3}{2} \quad \text{sub in } \textcircled{2}$$

$$4 + 2s = 8 + \frac{9}{2}$$

$$\Rightarrow s = \frac{17}{4}$$

Subbing $t = \frac{3}{2}$ and $s = \frac{17}{4}$ into $\textcircled{1}$ gives:

$$1 - \frac{17}{4} = 2 + \frac{3}{2}$$

$$-\frac{13}{4} = \frac{7}{2} \quad \text{Not equal } \therefore \text{ A and B skew}$$

(ii) Direction vector of A = $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, C = $\begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$

Since direction vector of A $\times -2$ = direction vector of C, the two lines are parallel

$$7. (i) \quad x = 2 \sin t \quad \frac{dx}{dt} = 2 \cos t$$

$$y = \cos 2t + 2 \sin t \quad \frac{dy}{dt} = -2 \sin 2t + 2 \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{-2 \sin 2t + 2 \cos t}{2 \cos t}$$

$$= \frac{\cancel{2}(-2 \sin t \cos t + \cos t)}{\cancel{2} \cos t} = \frac{\cos t(1 - 2 \sin t)}{\cos t}$$

$$= 1 - 2 \sin t$$

$$\frac{dy}{dx} = 0$$

$$1 - 2 \sin t = 0$$

$$\sin t = \frac{1}{2}$$

$$t = \frac{\pi}{6}$$

$$x = 2 \sin \frac{\pi}{6} = 1$$

$$y = \cos \frac{2\pi}{6} + 2 \sin \frac{\pi}{6} = \frac{3}{2}$$

\therefore stationary point at $(1, \frac{3}{2})$

Don't forget to
put this on
your sketch
in part (iii)

$$7(ii) \quad x = 2 \sin t \quad \therefore \frac{x}{2} = \sin t$$

$$y = \cos 2t + 2 \sin t \quad (\text{Use } \cos 2\theta = 1 - 2\sin^2\theta)$$

$$y = 1 - 2\sin^2 t + 2\sin t$$

$$y = 1 - 2\left(\frac{x}{2}\right)^2 + x$$

$$y = -\frac{x^2}{2} + x + 1$$

$$(iii) \quad -2 \leq x \leq 2$$

To find roots

$$-\frac{x^2}{2} + x + 1 = 0$$

$$\frac{x^2}{2} - x - 1 = 0$$

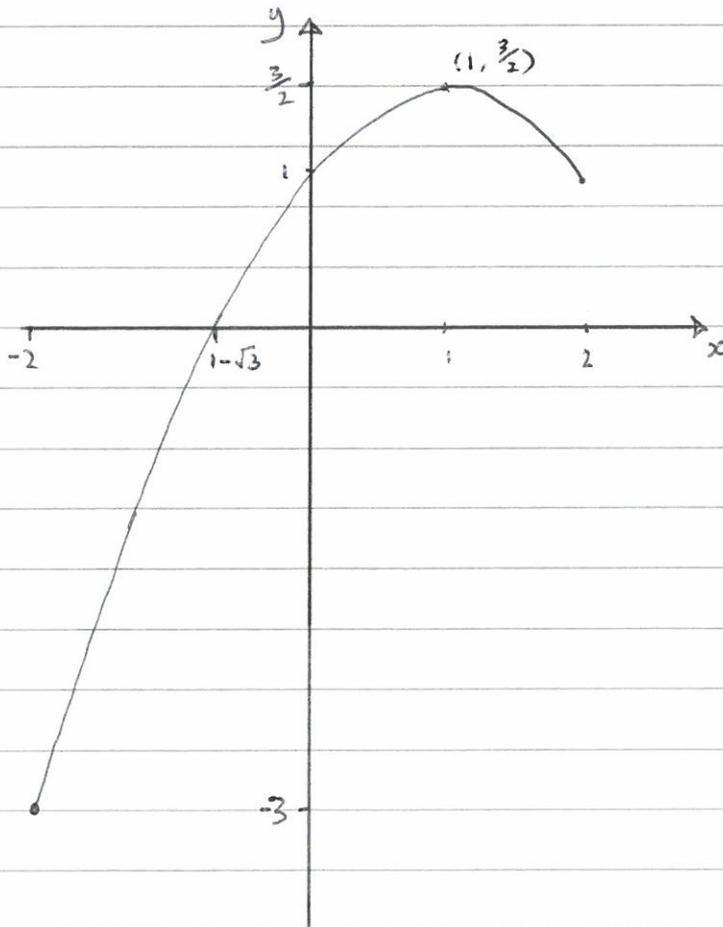
$$x^2 - 2x - 2 = 0$$

$$(x-1)^2 - 3 = 0$$

$$(x-1)^2 = 3$$

$$x = 1 \pm \sqrt{3}$$

$$x = 1 - \sqrt{3} //$$



$$8. (i) \begin{array}{r} t^2 - 2t + 4 \\ t+2 \overline{) t^3 + 0t^2 + 0t + 0} \\ \underline{t^3 + 2t^2} \\ -2t^2 + 0t \\ \underline{-2t^2 - 4t} \\ 4t + 0 \\ \underline{4t + 8} \\ -8 \end{array}$$

$$\therefore \frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}$$

$$(ii) \int_1^2 6t^2 \ln(t+2) dt \quad \text{by parts} \quad u = \ln(t+2) \quad u' = \frac{1}{t+2}$$

$$v' = 6t^2 \quad v = 2t^3$$

$$I = \left[2t^3 \ln(t+2) - 2 \int \frac{t^3}{t+2} dt \right]_1^2 \quad \text{Now use part (i)}$$

$$= \left[2t^3 \ln(t+2) - 2 \int t^2 - 2t + 4 - \frac{8}{t+2} dt \right]_1^2$$

$$= \left[2t^3 \ln(t+2) - 2 \left(\frac{t^3}{3} - t^2 + 4t - 8 \ln(t+2) \right) \right]_1^2$$

$$= \left(16 \ln 4 - 2 \left(\frac{8}{3} - 4 + 8 - 8 \ln 4 \right) \right) - \left(2 \ln 3 - 2 \left(\frac{1}{3} - 1 + 4 - 8 \ln 3 \right) \right)$$

$$= 32 \ln 4 - \frac{40}{3} - \left(18 \ln 3 - \frac{20}{3} \right) = -\frac{20}{3} - 18 \ln 3 + 32 \ln 4$$

$$9. \quad \frac{2+x^2}{(1+2x)(1-x)^2} = \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

$$2+x^2 = A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$$

$$\text{let } x=1 \quad 3=3C \Rightarrow C=1$$

$$x=-\frac{1}{2} \quad \frac{9}{4} = \frac{9}{4}A \Rightarrow A=1$$

$$x=0 \quad 2 = A+B+C \Rightarrow B=0$$

$$\therefore \frac{2+x^2}{(1+2x)(1-x)^2} = \frac{1}{1+2x} + \frac{1}{(1-x)^2}$$

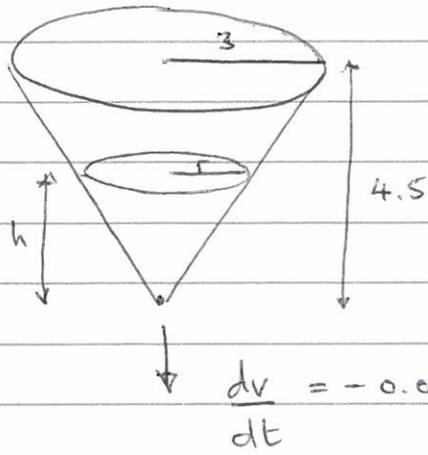
$$\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2} dx = \int_0^{\frac{1}{4}} \frac{1}{1+2x} + (1-x)^{-2} dx$$

$$= \left[\frac{1}{2} \ln(1+2x) + (1-x)^{-1} \right]_0^{\frac{1}{4}}$$

$$= \left(\frac{1}{2} \ln\left(\frac{3}{2}\right) + \frac{4}{3} \right) - \left(\ln 1 + 1 \right)$$

$$= \frac{1}{2} \ln \frac{3}{2} + \frac{1}{3} \quad \text{as required.}$$

10 (i)



$$\text{Note: } \frac{r}{h} = \frac{3}{4.5}$$

$$\therefore r = \frac{2h}{3}$$

$$\frac{dv}{dt} = -0.01 = -\frac{1}{100}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{2h}{3}\right)^2 h = \frac{4h^3 \pi}{27}$$

$$\frac{dv}{dh} = \frac{4 \times 3 h^2 \pi}{27} = \frac{4\pi h^2}{9}$$

$$\frac{dh}{dt} = \frac{dv}{dt} \cdot \frac{dh}{dv} = -\frac{1}{100} \cdot \frac{9}{4\pi h^2}$$

$$\frac{dh}{dt} = \frac{-9}{400\pi h^2} \quad \text{as required}$$

$$(ii) \quad h^2 \frac{dh}{dt} = -\frac{9}{400\pi}$$

$$\int h^2 dh = \int \frac{-9}{400\pi} dt$$

$$\frac{h^3}{3} = \frac{-9t}{400\pi} + C$$

$$\text{when } t=0 \quad h = 4.5 = \frac{9}{2}$$

$$\frac{\left(\frac{9}{2}\right)^3}{3} = C \quad \therefore C = \frac{243}{8}$$

$$\frac{h^3}{3} = \frac{-9t}{400\pi} + \frac{243}{8}$$

$$h = \sqrt[3]{\frac{-27t}{400\pi} + \frac{729}{8}}$$

$$(iii) \quad h=0 \quad \frac{-27t}{400\pi} + \frac{729}{8} = 0$$

$$t = \frac{400\pi \times -729}{-27 \times 8} = 1350\pi$$

$$\text{time to empty} = \frac{1350\pi}{60} \text{ mins} = 71 \text{ mins}$$