

June 08

$$5. a) \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} \cdot \tan\alpha = 8$$

$$2\tan^2\alpha = 8 - 8\tan^2\alpha$$

$$10\tan^2\alpha = 8$$

$$\tan^2\alpha = \frac{4}{5}$$

$$\tan\alpha = \frac{2}{\sqrt{5}} \quad \text{or} \quad \tan\alpha = -\frac{2}{\sqrt{5}}$$

$$\alpha = 41.8^\circ \checkmark$$

$$\alpha = 138.2^\circ \checkmark$$

$$b) (i) \sin\beta = \frac{6}{7} \Rightarrow \operatorname{cosec}\beta = \frac{7}{6} \checkmark$$

$$(ii) 1 + \cot^2\beta = \operatorname{cosec}^2\beta$$

$$\cot^2\beta = \frac{49}{36} - 1 = \frac{13}{36} \checkmark$$

$$8.1) T(\theta) = 3 [\cos\theta \cos 60 + \sin\theta \sin 60] + 2 [\cos\theta \cos 60 - \sin\theta \sin 60]$$

$$= \frac{3 \cos\theta + 3\sqrt{3} \sin\theta + \cos\theta - \sqrt{3} \sin\theta}{2}$$

$$= \frac{5 \cos\theta + \sqrt{3} \sin\theta}{2}$$

$$T(\theta) = \frac{\sqrt{3}}{2} \sin\theta + \frac{5}{2} \cos\theta \quad \checkmark \quad A = \frac{\sqrt{3}}{2} \quad B = \frac{5}{2}$$

$$ii) \frac{\sqrt{3}}{2} \sin\theta + \frac{5}{2} \cos\theta = R \sin(\theta + \alpha)$$

$$= R [\sin\theta \cos\alpha + \cos\theta \sin\alpha]$$

$$= R \cos\alpha \sin\theta + R \sin\alpha \cos\theta$$

equate coefficients

$$(i) \frac{\sqrt{3}}{2} = R \cos\alpha \quad (ii) \frac{5}{2} = R \sin\alpha$$

$$R^2 = \frac{3}{4} + \frac{25}{4} = 7 \quad \therefore R = \sqrt{7}$$

$$ii) \frac{\tan\alpha}{i} = \frac{5}{\sqrt{3}} \Rightarrow \alpha = 70.9^\circ$$

$$= \sqrt{7} \sin(\theta + 70.9^\circ) \quad \checkmark$$

$$(iii) \sqrt{7} \sin(\theta + 70.9^\circ) + 1 = 0$$

$$\sin(\theta + 70.9^\circ) = \frac{-1}{\sqrt{7}}$$

$$\theta + 70.9^\circ = \sin^{-1}\left(\frac{-1}{\sqrt{7}}\right) = -22.2^\circ, 202.2^\circ$$

$$\theta = 202.2^\circ - 70.9^\circ = 131.3^\circ \quad \checkmark$$

June 2009

1. (i) Fig 1  $y = \sec x$  (ii) Fig 2  $y = \cot x$  (iii)  $y = \sin^{-1} x$

3. (i)  $1 + \tan^2 A = \sec^2 A$  (use this)

$$1 + \tan^2 \alpha - (1 + \tan^2 \beta) = 16$$

$$1 + (m+2)^2 - 1 - m^2 = 16$$

$$\cancel{1} + \cancel{m^2} + 4m + 4 - \cancel{1} - \cancel{m^2} = 16$$

$$4m = 12$$

$$m = 3 \quad \checkmark$$

$$(ii) \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{5 + 3}{1 - 5 \times 3} = \frac{8}{-14} = -\frac{4}{7} \quad \checkmark$$

$$\begin{aligned}
 7. \quad i) \quad 8 \sin \theta - 6 \cos \theta &= R \sin(\theta - \alpha) \\
 &= R [\sin \theta \cos \alpha - \cos \theta \sin \alpha] \\
 &= R \sin \theta \cos \alpha - R \cos \theta \sin \alpha
 \end{aligned}$$

$$(i) \quad 8 = R \cos \alpha$$

$$(ii) \quad 6 = R \sin \alpha$$

$$R^2 = 8^2 + 6^2 \quad \therefore R = 10 \quad \checkmark$$

$$\frac{(ii)}{(i)} \quad \tan \alpha = \frac{3}{4} \quad \alpha = 36.9^\circ, \quad (36.8699\dots)$$

$$= 10 \sin(\theta - 36.9^\circ)$$

$$\begin{aligned}
 ii) a) \quad 10 \sin(\theta - 36.9^\circ) &= 9 \\
 \sin(\theta - 36.9^\circ) &= 0.9
 \end{aligned}$$

$$\theta - 36.9^\circ = \sin^{-1}(0.9) = 64.16^\circ, 115.84^\circ$$

$$\theta = 101.0^\circ, 152.7^\circ \quad (1 \text{ dp})$$

$$b) \quad 32 \sin x - 24 \cos x - (16 \sin y - 12 \cos y)$$

$$4[8 \sin x - 6 \cos x] - 2[8 \sin y - 6 \cos y]$$

$$\begin{aligned}
 &= \underbrace{40 \sin(x - 36.9^\circ)}_{\text{greatest} = 1} - 20 \underbrace{\sin(y - 36.9^\circ)}_{\text{least} = -1}
 \end{aligned}$$

$$\text{greatest value} = 40 - (-20) = 60 \quad \checkmark$$

Jan 10

2. (i)  $6 \sin 2\theta = 5 \cos \theta$

$$6 \times 2 \sin \theta \cos \theta = 5 \cos \theta$$

$$12 \sin \theta \cos \theta - 5 \cos \theta = 0$$

$$\cos \theta (12 \sin \theta - 5) = 0$$

$$\cos \theta = 0 \quad \sin \theta = \frac{5}{12} \quad \checkmark$$

(ii)  $8 \cos \theta \cos^2 \theta = 3$

$$8 \cos \theta \frac{1}{\sin^2 \theta} = 3$$

$$8 \cos \theta = 3 \sin^2 \theta \quad \Rightarrow \quad 8 \cos \theta = 3(1 - \cos^2 \theta)$$

$$8 \cos \theta = 3 - 3 \cos^2 \theta$$

$$3 \cos^2 \theta + 8 \cos \theta - 3 = 0$$

$$\cos \theta = \frac{-8 \pm \sqrt{64 + 36}}{6} = \frac{-8 \pm 10}{6} = -3 \text{ or } \frac{1}{3}$$

Since  $\theta$  is acute  $\cos \theta = \frac{1}{3}$  ✓ [Note  $\cos \theta = -3$  is undefined]

$$9 \text{ (i) } \tan 55 = \tan(45+10) = \frac{\tan 45 + \tan 10}{1 - \tan 45 \tan 10}$$

$$= \frac{1 + p}{1 - p} \quad \checkmark$$

$$\text{(ii) } \tan 5 = \tan(60-55) = \frac{\tan 60 - \tan 55}{1 + \tan 60 \tan 55}$$

$$= \frac{\sqrt{3} - \frac{1+p}{1-p}}{1 + \sqrt{3} \left( \frac{1+p}{1-p} \right)} = \frac{\sqrt{3}(1-p) - (1+p)}{1-p}$$

$$= \frac{\sqrt{3} - \sqrt{3}p - 1 - p}{1-p + \sqrt{3} + \sqrt{3}p}$$

Not very pretty but correct.

$$= \frac{\sqrt{3}(1-p) - (1+p)}{(1-p) + \sqrt{3}(1+p)} = \frac{\sqrt{3} - \sqrt{3}p - 1 - p}{1-p + \sqrt{3} + \sqrt{3}p}$$

See next sheets for more attractive answers.

$$= \frac{\sqrt{3} - 1 - p(\sqrt{3} + 1)}{1 + \sqrt{3} + p(\sqrt{3} - 1)}$$

$$3 \tan \theta + 3 \tan 10 = 7 + \tan 10 \tan \theta$$

$$3 \tan \theta + 3p = 7 + p \tan \theta$$

$$\tan \theta (3 - p) = 7 - 3p$$

$$\tan \theta = \frac{7-3p}{3-p}$$

$$\frac{3-7p}{7-p-3}$$

$$\text{(iii) } 3 \sin(\theta+10) = 7 \cos(\theta-10^\circ)$$

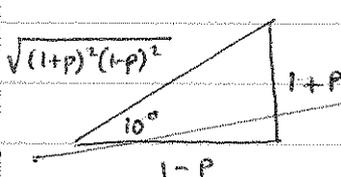
$$3 [\sin \theta \cos 10 + \cos \theta \sin 10] = 7 [\cos \theta \cos 10 + \sin \theta \sin 10]$$

$$\frac{3 \cos 10 \sin \theta}{\cos 10 \cos \theta} + \frac{3 \sin 10 \cos \theta}{\cos 10 \cos \theta} = \frac{7 \cos 10 \cos \theta}{\cos 10 \cos \theta} + \frac{7 \sin 10 \sin \theta}{\cos 10 \cos \theta}$$

$$\sin \theta (3 \cos 10 - 7 \sin 10) = \cos \theta (7 \cos 10 - 3 \sin 10)$$

$$\tan \theta = \frac{7 \cos 10 - 3 \sin 10}{3 \cos 10 - 7 \sin 10} \div \cos 10 = \frac{7 - 3p}{3 - 7p}$$

Bit of a horror show!  
- see later sheet!



$$\tan \theta = \frac{1-p}{\sqrt{(1+p)^2 + (1-p)^2}}$$

see next sheet

$$9(\text{iii}) \quad \tan S = \tan(10-S) = \frac{\tan 10 - \tan S}{1 + \tan 10 \tan S}$$

$$\tan S = \frac{p - \tan S}{1 + p \tan S} \quad (\text{now make } \tan S \text{ the subject,})$$

$$\tan S + p \tan^2 S = p - \tan S$$

$$p \tan^2 S + 2 \tan S - p = 0 \quad \text{use formula.}$$

$$\tan S = \frac{-2 \pm \sqrt{4 + 4p^2}}{2p} = \frac{-2 \pm \sqrt{4(1+p^2)}}{2p}$$

$$= \frac{-2 \pm 2\sqrt{1+p^2}}{2p} = \frac{-1 \pm \sqrt{1+p^2}}{p}$$

Since  $\tan 10$  positive,  $p$  is positive

$$\text{Since } \tan S \text{ is positive } \tan S = \frac{-1 + \sqrt{1+p^2}}{p}$$

2 (ii)  
9 (ii)

$$\tan 10 = \tan(2 \times 5) = \frac{2 \tan 5}{1 - \tan^2 5}$$

$$p = \frac{2 \tan 5}{1 - \tan^2 5}$$

$$p - p \tan^2 5 = 2 \tan 5$$

$$p \tan^2 5 + 2 \tan 5 - p = 0$$

use formula

$$\begin{aligned} \tan 5 &= \frac{-2 \pm \sqrt{4 + 4p^2}}{2p} \\ &= \frac{-2 \pm \sqrt{4(1+p^2)}}{2p} \\ &= \frac{-2 \pm 2\sqrt{(1+p^2)}}{2p} \\ &= \frac{-1 \pm \sqrt{(1+p^2)}}{p} \end{aligned}$$

Since  $\tan 10$  is positive,  $p$  is positive

Since  $\tan 5$  is positive

$$\tan 5 = \frac{-1 + \sqrt{(1+p^2)}}{p}$$

$$9 \text{ (iii)} \quad 3 \sin(\theta + 10) = 7 \cos(\theta - 10)$$

$$3 [\sin \theta \cos 10 + \cos \theta \sin 10] = 7 [\cos \theta \cos 10 + \sin \theta \sin 10]$$

$$3 \sin \theta \cos 10 + 3 \cos \theta \sin 10 = 7 \cos \theta \cos 10 + 7 \sin \theta \sin 10$$

÷ by  $\cos \theta \cos 10$  (this takes some spotting!)

$$\frac{3 \cancel{\sin \theta \cos 10}}{\cancel{\cos \theta \cos 10}} + \frac{3 \cancel{\cos \theta \sin 10}}{\cancel{\cos \theta \cos 10}} = \frac{7 \cancel{\cos \theta \cos 10}}{\cancel{\cos \theta \cos 10}} + \frac{7 \sin \theta \sin 10}{\cos \theta \cos 10}$$

$$3 \tan \theta + 3 \tan 10 = 7 + 7 \tan \theta \tan 10$$

$$3 \tan \theta + 3p = 7 + 7p \tan \theta$$

$$\tan \theta (3 - 7p) = 7 - 3p$$

$$\tan \theta = \frac{7 - 3p}{3 - 7p} \quad \text{or equally} \quad \frac{3p - 7}{7p - 3}$$

Jun 10

$$3 \quad (i) \quad \operatorname{cosec} \theta (3 \cos 2\theta + 7) + 11 = 0$$

$$\frac{1}{\sin \theta} (3(1 - 2\sin^2 \theta) + 7) + 11 = 0$$

$$3(1 - 2\sin^2 \theta) + 7 + 11\sin \theta = 0$$

$$3 - 6\sin^2 \theta + 7 + 11\sin \theta = 0$$

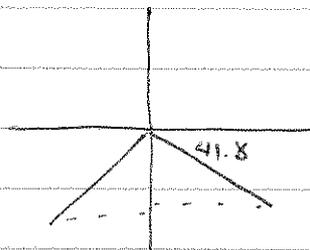
$$6\sin^2 \theta - 11\sin \theta - 10 = 0 \quad \checkmark$$

$$(ii) \quad \sin \theta = \frac{11 \pm \sqrt{121 + 240}}{12} = \frac{11 \pm \sqrt{361}}{12}$$

$$\sin \theta = \frac{11 \pm 19}{12} = \frac{30}{12}, \frac{-8}{12} = \frac{5}{2}, -\frac{2}{3}$$

$\sin \theta = \frac{5}{2}$  no real solutions

$$\sin \theta = -\frac{2}{3} \quad \theta = -41.8, -138.2^\circ \quad \checkmark$$



June 10

8. (i)  $3 \cos x + 3 \sin x = R \cos(x - \alpha)$

$$3 \cos x + 3 \sin x = R [\cos x \cos \alpha + \sin x \sin \alpha]$$

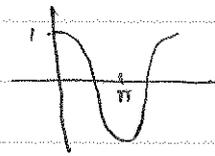
$$3 \cos x + 3 \sin x = R \cos \alpha \cos x + R \sin \alpha \sin x$$

(i)  $3 = R \cos \alpha$     ii  $3 = R \sin \alpha$

$$R^2 = 3^2 + 3^2 = 18 \quad \therefore R = \sqrt{18} = 3\sqrt{2} \checkmark$$

$$\tan \alpha = 1 \quad \alpha = \frac{\pi}{4} \checkmark$$

$$3\sqrt{2} \cos(x - \frac{\pi}{4}) //$$



(ii) a)  $T(x) = \frac{8}{3\sqrt{2} \cos(x - \frac{\pi}{4})}$     \* let denominator = 0

$$\therefore \cos(x - \frac{\pi}{4}) = 0$$

$$x - \frac{\pi}{4} = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} //$$

b)  $T(3x) = \frac{8}{3\sqrt{2} \cos(3x - \frac{\pi}{4})} = \frac{8\sqrt{6}}{9}$

$$9 = \sqrt{6} \cdot 3\sqrt{2} \cos(3x - \frac{\pi}{4})$$

$$\frac{\sqrt{3}}{2} = \cos(3x - \frac{\pi}{4})$$

$$3x - \frac{\pi}{4} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}, -\frac{\pi}{6}$$

$$3x = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi + 3\pi}{12} = \frac{5\pi}{12}$$

$$x = \frac{5\pi}{36} //$$

X

$$3x = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

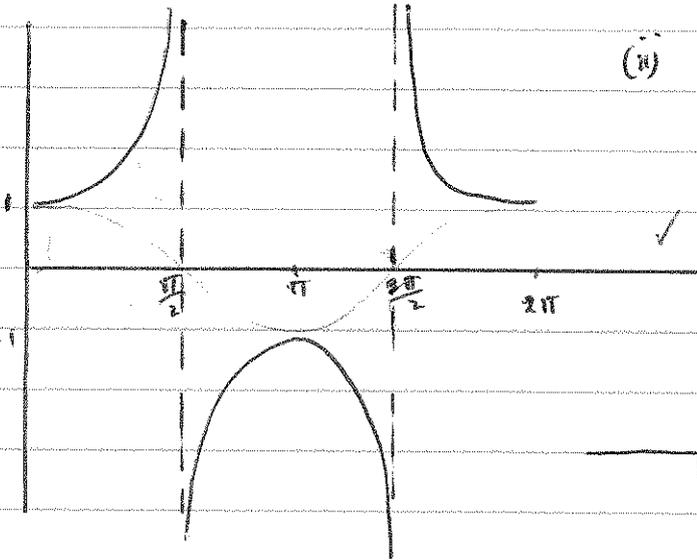
$$x = \frac{\pi}{36} \checkmark$$

THIS IS SMALLEST //

June 7

7.

(i)



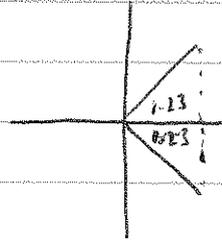
(ii)

$$\sec x = 3$$

$$\cos x = \frac{1}{3}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = 1.23^\circ, 5.05^\circ$$



(iii)

$$\sec \theta = 5 \sec \theta$$

$$\frac{1}{\cos \theta} = \frac{5}{\sin \theta} \Rightarrow \frac{\sin \theta}{\cos \theta} = 5 \Rightarrow \tan \theta = 5$$

$$\theta = \tan^{-1} 5 = 1.37^\circ, 4.51^\circ$$

June 7

9/

$$(i) \tan(\theta + 60^\circ) \tan(\theta - 60^\circ) = \frac{\tan^2 \theta - 3}{1 - 3\tan^2 \theta}$$

$$\text{LHS} = \frac{\tan \theta + \tan 60^\circ}{1 - \tan \theta \tan 60^\circ} \cdot \frac{\tan \theta - \tan 60^\circ}{1 + \tan \theta \tan 60^\circ}$$

$$= \frac{(\tan \theta + \sqrt{3})(\tan \theta - \sqrt{3})}{(1 - \sqrt{3}\tan \theta)(1 + \sqrt{3}\tan \theta)}$$

$$= \frac{\tan^2 \theta - 3}{1 - 3\tan^2 \theta} = \text{RHS} \quad \checkmark$$



$$(ii) \tan(\theta + 60^\circ) \tan(\theta - 60^\circ) = 4 \sec^2 \theta - 3 \quad \text{use part (i)}$$

$$\frac{\tan^2 \theta - 3}{1 - 3\tan^2 \theta} = 4(1 + \tan^2 \theta) - 3 \quad \text{using } 1 + \tan^2 \theta = \sec^2 \theta \quad \checkmark$$

$$\tan^2 \theta - 3 = 4(1 + \tan^2 \theta)(1 - 3\tan^2 \theta) - 3(1 - 3\tan^2 \theta)$$

$$\tan^2 \theta - 3 = 4(1 - 2\tan^2 \theta - 3\tan^4 \theta) - 3 + 9\tan^2 \theta$$

$$\tan^2 \theta - 3 = 4 - 8\tan^2 \theta - 12\tan^4 \theta - 3 + 9\tan^2 \theta$$

$$0 = 4 - 12\tan^4 \theta \Rightarrow 4 = 12\tan^4 \theta$$

$$1 = 3\tan^4 \theta \Rightarrow \tan^4 \theta = \frac{1}{3}$$

$$\tan \theta = \pm \sqrt[4]{\frac{1}{3}} \quad \theta = \tan^{-1} \sqrt[4]{\frac{1}{3}} = 37.2^\circ, 217.2^\circ$$

$$\theta = \tan^{-1} -\sqrt[4]{\frac{1}{3}} = 142.8^\circ \checkmark$$

$$9 \text{ (ii)} \quad \tan(\theta + 60) \tan(\theta - 60) = k^2$$

$$\frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta} = k^2$$

$$\tan^2 \theta - 3 = k^2(1 - 3 \tan^2 \theta)$$

$$\tan^2 \theta - 3 = k^2 - 3k^2 \tan^2 \theta$$

$$\tan^2 \theta (1 + 3k^2) = k^2 + 3$$

$$\tan^2 \theta = \frac{k^2 + 3}{1 + 3k^2}$$

Since  $k^2$  is always +ve this expression will be positive.

$$\tan \theta = \pm \sqrt{\frac{k^2 + 3}{1 + 3k^2}}$$

$$\theta = \arctan \sqrt{\frac{k^2 + 3}{1 + 3k^2}}$$

gives solution in 1<sup>st</sup> quadrant  
 $0 < \theta \leq 90$

$$\text{or } \theta = \arctan -\sqrt{\frac{k^2 + 3}{1 + 3k^2}}$$

gives solution in 2<sup>nd</sup> quadrant.  
 $90 < \theta \leq 180$

Jan 08

$$3. a) \sec \frac{1}{2} \alpha = 4$$

$$\frac{1}{\cos \frac{1}{2} \alpha} = 4 \Rightarrow \cos \frac{1}{2} \alpha = \frac{1}{4}$$

$$\frac{1}{2} \alpha = \cos^{-1} \frac{1}{4} = 75.5^\circ, \cancel{284.5^\circ}$$

$$\alpha = 151.0 \quad (\text{idp}) \quad \checkmark$$

$$b) \tan \beta = 7 \cot \beta$$

$$\tan \beta = \frac{7}{\tan \beta} \Rightarrow \tan^2 \beta = 7$$

$$\tan \beta = \pm \sqrt{7} \quad \beta = \tan^{-1} \sqrt{7} = 69.3^\circ \quad \checkmark$$

$$\beta = \tan^{-1} -\sqrt{7} = 110.7^\circ \quad \checkmark$$

Jawab

$$9. (i) 4 \cos(\theta + 60^\circ) \cos(\theta + 30^\circ) \equiv \sqrt{3} - 2 \sin 2\theta$$

$$\text{LHS} = 4 [\cos \theta \cos 60 - \sin \theta \sin 60] [\cos \theta \cos 30 - \sin \theta \sin 30]$$

$$= 4 \left[ \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \right] \left[ \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right]$$

$$= 4 \left[ \frac{\sqrt{3}}{4} \cos^2 \theta - \frac{1}{4} \sin \theta \cos \theta - \frac{3}{4} \sin \theta \cos \theta + \frac{\sqrt{3}}{4} \sin^2 \theta \right]$$

$$= 4 \left[ \frac{\sqrt{3}}{4} (\cos^2 \theta + \sin^2 \theta) - \sin \theta \cos \theta \right]$$

$$= \sqrt{3} - 4 \sin \theta \cos \theta$$

$$= \sqrt{3} - 2 \sin 2\theta = \text{RHS} \quad \checkmark$$

$$(ii) 4 \cos 82.5 \cos 52.5$$

$$= 4 \cos(22.5 + 60) \cos(22.5 + 30)$$

$$= \sqrt{3} - 2 \sin 45 = \sqrt{3} - 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{3} - \frac{2\sqrt{2}}{\sqrt{2}} = \sqrt{3} - \sqrt{2} \quad \checkmark$$

$$(iii) 4 \cos(\theta + 60) \cos(\theta + 30) = 1$$

$$\sqrt{3} - 2 \sin 2\theta = 1$$

$$2 \sin 2\theta = \sqrt{3} - 1$$

$$\sin 2\theta = \frac{\sqrt{3} - 1}{2} \Rightarrow 2\theta = \sin^{-1} \left( \frac{\sqrt{3} - 1}{2} \right)$$

$$2\theta = 21.47, 158.53, \dots$$

$$\theta = 10.7^\circ, 79.3^\circ \text{ idp.}$$

$$(iv) \sqrt{3} - 2 \sin 2\theta = k$$

$$2 \sin 2\theta = \sqrt{3} - k$$

$$\sin 2\theta = \frac{\sqrt{3} - k}{2}$$

$$\frac{\sqrt{3} - k}{2} > 1$$

$$k < \sqrt{3} - 2 \quad \checkmark$$

$$\text{or } \frac{\sqrt{3} - k}{2} < -1$$

$$k > \sqrt{3} + 2 \quad \checkmark$$