JUNE 2008

5 **(a)** Express $\tan 2\alpha$ in terms of $\tan\alpha$ and hence solve, for $0^\circ < \alpha < 180^\circ$, the equation

$$\tan 2\alpha \tan\alpha = 8.$$ [6]

(b) Given that β is the acute angle such that $\sin\beta = \frac{6}{7}$, find the exact value of

(i) $\operatorname{cosec}\beta$, [1]

(ii) $\cot^2\beta$. [2]

8 The expression $\mathrm{T}(\theta)$ is defined for θ in degrees by

$$\mathrm{T}(\theta) = 3\cos(\theta - 60^\circ) + 2\cos(\theta + 60^\circ).$$

(i) Express $\mathrm{T}(\theta)$ in the form $A\sin\theta + B\cos\theta$, giving the exact values of the constants A and B. [3]

(ii) Hence express $\mathrm{T}(\theta)$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the smallest positive value of θ such that $\mathrm{T}(\theta) + 1 = 0$. [4]

June 2009

1

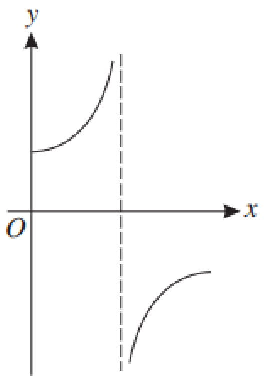

Fig. 1

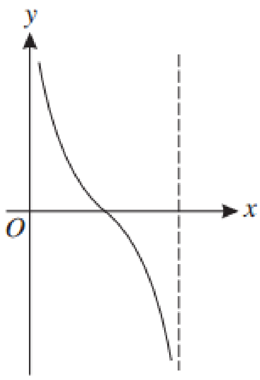

Fig. 2

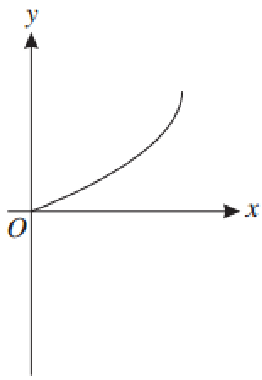

Fig. 3

Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

State which equation corresponds to

(i) Fig. 1, **[1]**

(ii) Fig. 2, **[1]**

(iii) Fig. 3. **[1]**

3 The angles α and β are such that

$$\tan\alpha = m + 2 \quad \text{and} \quad \tan\beta = m,$$

where m is a constant.

(i) Given that $\sec^2\alpha - \sec^2\beta = 16$, find the value of m. **[3]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

7 **(i)** Express $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve, for $0° < \theta < 360°$, the equation $8\sin\theta - 6\cos\theta = 9$, **[4]**

(b) find the greatest possible value of

$$32\sin x - 24\cos x - (16\sin y - 12\cos y)$$

as the angles x and y vary. **[3]**

January 2010

2 The angle θ is such that $0° < \theta < 90°$.

(i) Given that θ satisfies the equation $6\sin 2\theta = 5\cos\theta$, find the exact value of $\sin\theta$. **[3]**

(ii) Given instead that θ satisfies the equation $8\cos\theta\operatorname{cosec}^2\theta = 3$, find the exact value of $\cos\theta$. **[5]**

9 The value of $\tan 10°$ is denoted by p. Find, in terms of p, the value of

(i) $\tan 55°$, **[3]**

(ii) $\tan 5°$, **[4]**

* **(iii)** $\tan\theta$, where θ satisfies the equation $3\sin(\theta + 10°) = 7\cos(\theta - 10°)$. **[5]**

June 2010

3 **(i)** Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. **[3]**

(ii) Hence solve, for $-180° < \theta < 180°$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$. **[3]**

8 **(i)** Express $3\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) The expression $\mathrm{T}(x)$ is defined by $\mathrm{T}(x) = \dfrac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which $\mathrm{T}(x)$ is not defined. **[2]**

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. **[4]**

JUNE 2007

7 **(i)** Sketch the graph of $y = \sec x$ for $0 \leqslant x \leqslant 2\pi$. [2]

(ii) Solve the equation $\sec x = 3$ for $0 \leqslant x \leqslant 2\pi$, giving the roots correct to 3 significant figures. [3]

(iii) Solve the equation $\sec\theta = 5\,\mathrm{cosec}\,\theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving the roots correct to 3 significant figures. [4]

9 **(i)** Prove the identity

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) \equiv \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}. \qquad [4]$$

(ii) Solve, for $0^\circ < \theta < 180^\circ$, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = 4\sec^2\theta - 3,$$

giving your answers correct to the nearest 0.1°. [5]

* **(iii)** Show that, for all values of the constant k, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = k^2$$

has two roots in the interval $0^\circ < \theta < 180^\circ$. [3]

JANUARY 2008

3 **(a)** Solve, for $0^\circ < \alpha < 180^\circ$, the equation $\sec\frac{1}{2}\alpha = 4$. [3]

(b) Solve, for $0^\circ < \beta < 180^\circ$, the equation $\tan\beta = 7\cot\beta$. [4]

9 **(i)** Use the identity for $\cos(A + B)$ to prove that

$$4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) \equiv \sqrt{3} - 2\sin 2\theta. \qquad [4]$$

(ii) Hence find the exact value of $4\cos 82.5^\circ \cos 52.5^\circ$. [2]

(iii) Solve, for $0^\circ < \theta < 90^\circ$, the equation $4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) = 1$. [3]

(iv) Given that there are no values of θ which satisfy the equation

$$4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) = k,$$

determine the set of values of the constant k. [3]