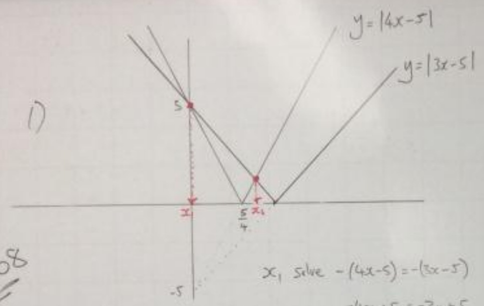
C3 June 2008 solutions by Mr Ely

C3 June 2008

1)



x_1 solve $-(4x-5) = -(3x-5)$

$-4x+5 = -3x+5$

$0 = x$ obvious!

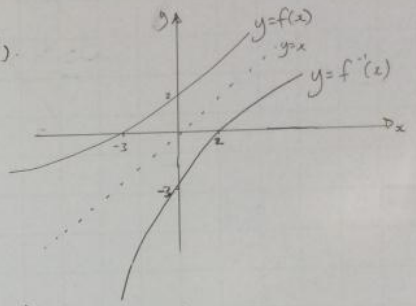
x_2 Solve $4x-5 = -(3x-5)$

$4x-5 = -3x+5$

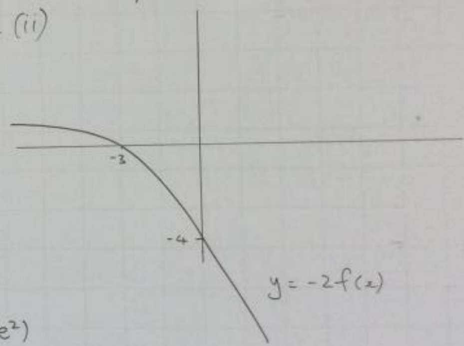
$7x = 10$

$x = \frac{10}{7}$

2 (i)



2 (ii)



3. $y = \overset{u}{x^2} \cdot \overset{v}{\ln x}$ (It's a product)

$\frac{dy}{dx} = \overset{u'v}{2x\ln x} + \overset{v'u}{\frac{1}{x}\cdot x^2}$ Product rule $= 2x\ln x + x = x(2\ln x + 1)$

when $x = e$ $\frac{dy}{dx} = e(2\ln e + 1) = 3e$ (Note $\ln e = 1$)

when $x = e$ $y = e^2 \ln e = e^2$ $\therefore$ point is at (e, e^2)

tangent: $y = mx + c$ so $y = 3ex + c$ sub in (e, e^2)

$e^2 = 3e^2 + c$

$c = -2e^2$ $\Longrightarrow$ $y = 3ex - 2e^2$

Jun '08 C3

4. (i) $y = (2x^2+9)^{\frac{5}{2}}$

$\frac{dy}{dx} = \frac{5}{2}(2x^2+9)^{\frac{3}{2}} \cdot 4x = 10x(2x^2+9)^{\frac{3}{2}}$

at P

$10x(2x^2+9)^{\frac{3}{2}} = 100$

$x = 10 \cdot \frac{1}{(2x^2+9)^{\frac{3}{2}}}$

$\therefore$ $x = 10(2x^2+9)^{-\frac{3}{2}}$

(ii) $f'(x) = 10x(2x^2+9)^{\frac{3}{2}}$

$f'(0.3) = 10 \times 0.3(2\times(0.3)^2+9)^{\frac{3}{2}} = 83.44$ (2dp)

$f'(0.4) = 10 \times 0.4(2\times(0.4)^2+9)^{\frac{3}{2}} = 113.81$ (2dp)

gradient changes from 83.44 to 113.81 so the value of x at P must be between 0.3 and 0.4

(iii) $x_{n+1} = 10(2x_n^2+9)^{-\frac{3}{2}}$

$x_0 = 0.35$ (your choice $x_0 = 0.3$ or 0.4 also sensible)

$x_1 = 10(2\times 0.35^2+9)^{-\frac{3}{2}} = 0.355746$

$x_2 = 0.355278$

$x_3 = 0.355316$

$x_4 = 0.355313$

$x = 0.3553$ 4dp

5. (a) $\tan 2\alpha = \tan(\alpha+\alpha)$

$= \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha \cdot \tan\alpha} = \frac{2\tan\alpha}{1-\tan^2\alpha}$ // ← You may just know this!

$\tan 2\alpha \tan\alpha = 8$

$\frac{2\tan\alpha}{1-\tan^2\alpha} \cdot \tan\alpha = 8$

$\frac{2\tan^2\alpha}{1-\tan^2\alpha} = 8$

$2\tan^2\alpha = 8 - 8\tan^2\alpha$

$10\tan^2\alpha = 8$

$\tan^2\alpha = \frac{4}{5}$

$\tan\alpha = \pm\frac{2}{\sqrt{5}}$

$\alpha = \tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\alpha = 41.8°$ other solutions outside $0<\alpha<180$

$\alpha = \tan^{-1}\left(-\frac{2}{\sqrt{5}}\right)$

$\alpha = -41.8° = 180 - 41.8° = 138.2°$

$\alpha = 41.8°$ and $138.2°$

5 (b) $\sin\beta = \frac{6}{7}$

(i) $\operatorname{cosec}\beta = \frac{1}{\sin\beta} = \frac{7}{6}$

(ii) $1 + \cot^2\beta = \operatorname{cosec}^2\beta$

$\cot^2\beta = \left(\frac{7}{6}\right)^2 - 1 = \frac{13}{36}$

C3 June 2008 solutions by Mr Ely

Jun 08 C3

6. $V = \pi\int y^2\,dx = \pi\int_0^{\frac{1}{2}} e^{6x}\,dx - \pi\int_0^{\frac{1}{2}}(2x-1)^8\,dx = \pi\int_0^{\frac{1}{2}} e^{6x} - (2x-1)^8\,dx$

$= \pi\left[\frac{1}{6}e^{6x} - \frac{1}{18}(2x-1)^9\right]_0^{\frac{1}{2}} = \pi\left[\left(\frac{1}{6}e^3 - 0\right) - \left(\frac{1}{6} + \frac{1}{18}\right)\right]$

$= \pi\left[\frac{1}{6}e^3 - \frac{2}{9}\right] = \pi\left[\frac{3e^3 - 4}{18}\right] = \left(\frac{3e^3-4}{18}\right)\pi$ or equivalent

e.g $\pi\left(\frac{e^3}{6} - \frac{2}{9}\right)$

Jun 08 C3

7. (i) When $t=0$ $N=42$

when $t=9$ $N=84$

Using 1st equation

$42 = A \times 2^0$

$\therefore \boxed{A = 42}$

$84 = 42 \times 2^{9k}$

$2 = 2^{9k}$

$1 = 9k$

$\boxed{k = \frac{1}{9}}$

Using 2nd equation

$84 = 42e^{9m}$

$2 = e^{9m}$

$\ln 2 = 9m$

$\boxed{m = \frac{1}{9}\ln 2}$

(ii) $100 = 42e^{\frac{1}{9}\ln 2 . t}$

$\frac{100}{42} = e^{\frac{1}{9}\ln 2 . t}$

$\ln\left(\frac{100}{42}\right) = \frac{1}{9}\ln 2 . t$

$t = \frac{9\ln\left(\frac{100}{42}\right)}{\ln 2} = 11.3$ years 3.s.f.

(iii) $N = 42e^{\frac{1}{9}\ln 2 . t}$

$\frac{dN}{dt} = 42.\frac{1}{9}\ln 2\, e^{\frac{1}{9}\ln 2 t}$

When $t=35$ $\frac{dN}{dt} = 42.\frac{1}{9}\ln 2\, e^{\frac{1}{9}\ln 2 . 35}$

$\frac{dN}{dt} = 47.9$ 3sf.

8. i) $T(\theta) = 3\left[\cos\theta\cos 60 + \sin\theta\sin 60\right] + 2\left[\cos\theta\cos 60 - \sin\theta\sin 60\right]$

$= 3\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] + 2\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right]$

$= \frac{5}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$

$A = \frac{\sqrt{3}}{2}$, $B = \frac{5}{2}$

ii) $\frac{\sqrt{3}}{2}\sin\theta + \frac{5}{2}\cos\theta = R\sin(\theta + \alpha)$

$= R\left[\sin\theta\cos\alpha + \cos\theta\sin\alpha\right]$

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

equating coefficients gives

(i) $\frac{\sqrt{3}}{2} = R\cos\alpha$ (ii) $\frac{5}{2} = R\sin\alpha$

$\frac{ii}{i}$ $\tan\alpha = \frac{5}{\sqrt{3}}$ $\therefore \alpha = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right) = 70.9^\circ$ 3sf

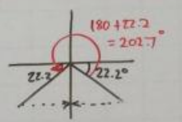
$(i)^2 + (ii)^2$ $\frac{3}{4} + \frac{25}{4} = R^2$ $\therefore R = \sqrt{7}$

$\sqrt{7}\sin(\theta + 70.9^\circ)$ //

8(iii) Using part (ii) $\sqrt{7}\sin(\theta + 70.9^\circ) + 1 = 0$

$\sin(\theta + 70.9^\circ) = -\frac{1}{\sqrt{7}}$

$\theta + 70.9^\circ = \sin^{-1}\left(-\frac{1}{\sqrt{7}}\right) = -22.2^\circ$, 202.2°



$\therefore \theta = 202.2^\circ - 70.9^\circ = 131.3^\circ$ 1dp. Need to work with unrounded values to get this to 1.d.p.

$= 131^\circ$ ← Accept answer to nearest degree

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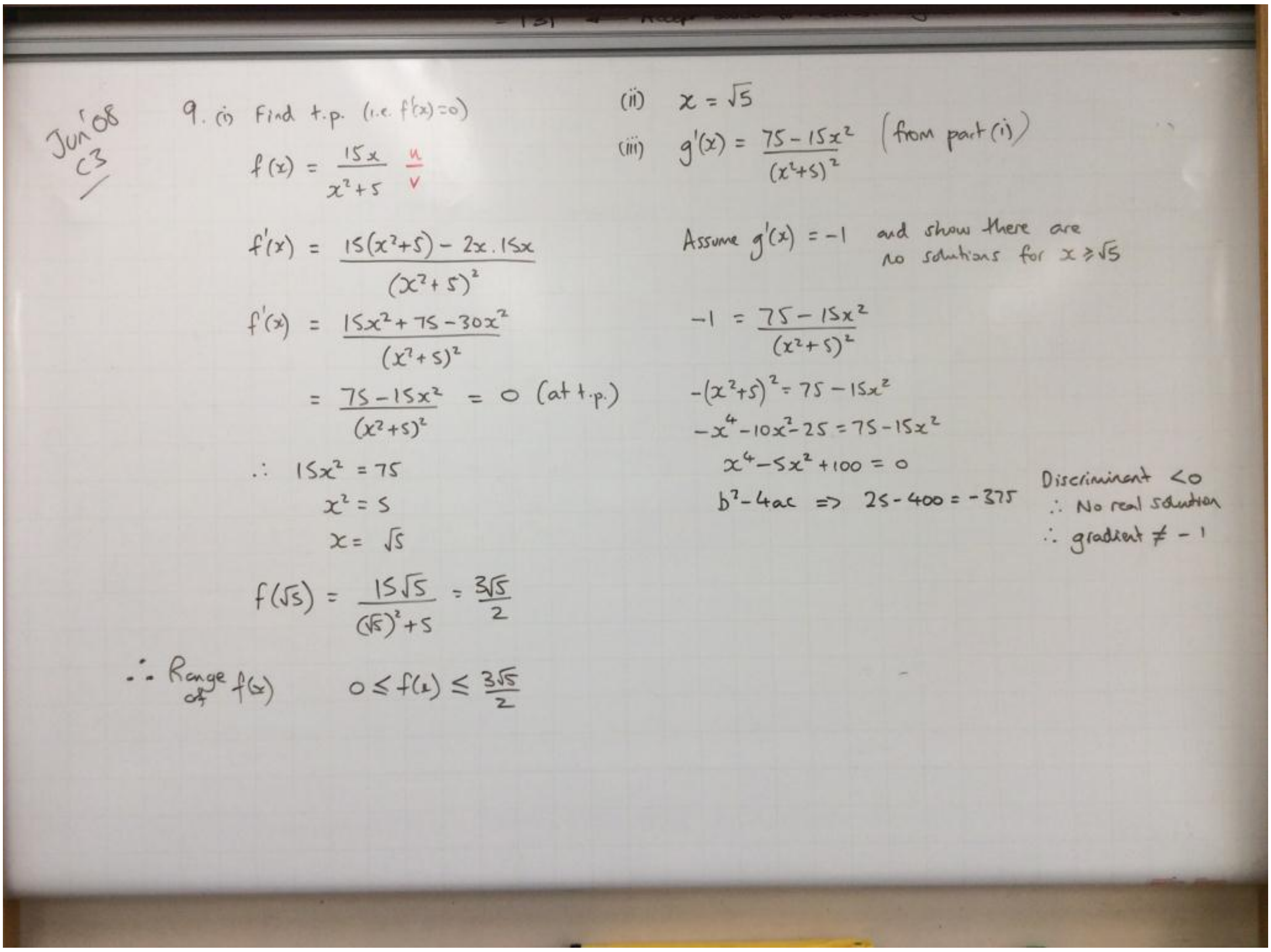

Jun '08 C3

9. (i) Find t.p. (i.e. $f'(x)=0$)

$$f(x) = \frac{15x}{x^2+5} \quad \frac{u}{v}$$

$$f'(x) = \frac{15(x^2+5) - 2x.15x}{(x^2+5)^2}$$

$$f'(x) = \frac{15x^2+75-30x^2}{(x^2+5)^2}$$

$$= \frac{75-15x^2}{(x^2+5)^2} = 0 \text{ (at t.p.)}$$

$\therefore$ $15x^2 = 75$

$x^2 = 5$

$x = \sqrt{5}$

$$f(\sqrt{5}) = \frac{15\sqrt{5}}{(\sqrt{5})^2+5} = \frac{3\sqrt{5}}{2}$$

$\therefore$ Range of $f(x)$ $\quad 0 \leq f(x) \leq \frac{3\sqrt{5}}{2}$

(ii) $x = \sqrt{5}$

(iii) $g'(x) = \frac{75-15x^2}{(x^2+5)^2}$ (from part (i))

Assume $g'(x) = -1$ and show there are no solutions for $x \geqslant \sqrt{5}$

$$-1 = \frac{75-15x^2}{(x^2+5)^2}$$

$-(x^2+5)^2 = 75 - 15x^2$

$-x^4 - 10x^2 - 25 = 75 - 15x^2$

$x^4 - 5x^2 + 100 = 0$

$b^2 - 4ac \Rightarrow 25 - 400 = -375$

Discriminant < 0

$\therefore$ No real solution

$\therefore$ gradient $\neq -1$