Jan 13

3 **(a)** Given that $|t| = 3$, find the possible values of $|2t - 1|$. **[3]**

(b) Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$. **[4]**

8 The functions f and g are defined for all real values of x by

$$f(x) = x^2 + 4ax + a^2 \quad \text{and} \quad g(x) = 4x - 2a,$$

where a is a positive constant.

(i) Find the range of f in terms of a. **[4]**

(ii) Given that $fg(3) = 69$, find the value of a and hence find the value of x such that $g^{-1}(x) = x$. **[6]**

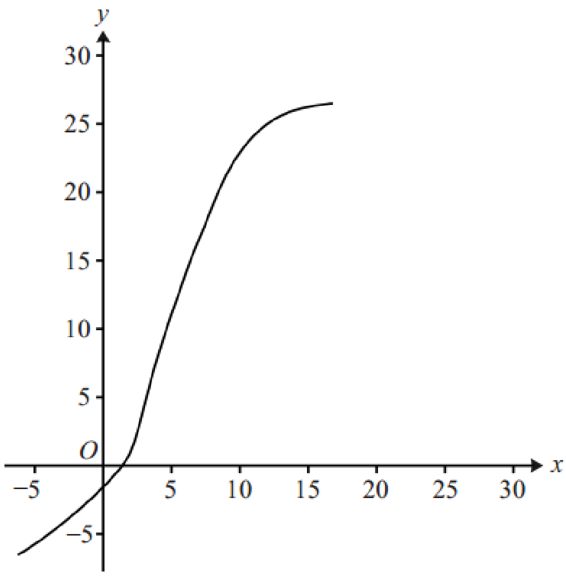
Jun 12

1 Solve the inequality $|2x - 5| > |x + 1|$. **[5]**

7 The function f is defined for all real values of x by $f(x) = 2x + 5$. The function g is defined for all real values of x and is such that $g^{-1}(x) = \sqrt[3]{x - a}$, where a is a constant. It is given that $fg^{-1}(12) = 9$. Find the value of a and hence solve the equation $gf(x) = 68$. **[7]**

Jan 12

5



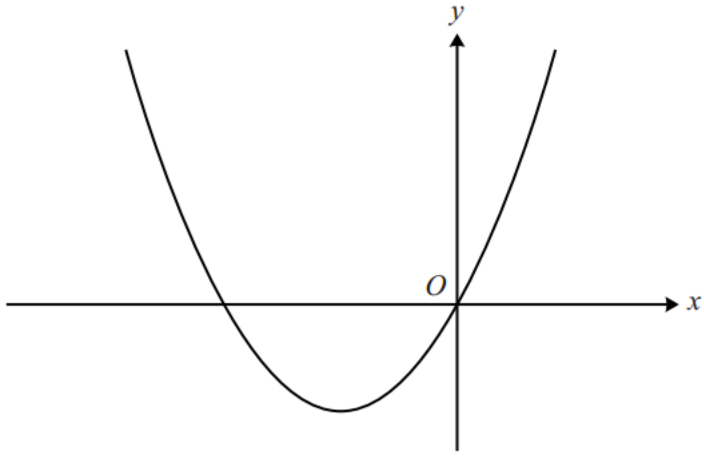
It is given that f is a one-one function defined for all real values. The diagram shows the curve with equation $y = f(x)$. The coordinates of certain points on the curve are shown in the following table.

x	2	4	6	8	10	12	14
y	1	8	14	19	23	25	26

(i) State the value of $ff(6)$ and the value of $f^{-1}(8)$. **[2]**

(ii) On the copy of the diagram, sketch the curve $y = f^{-1}(x)$, indicating how the curves $y = f(x)$ and $y = f^{-1}(x)$ are related. **[2]**

9



The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x),$$

where k is a positive constant. The diagram shows the curve with equation $y = f(x)$.

(i) The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations:
a translation parallel to the x-axis,
a translation parallel to the y-axis,
a stretch.
Give details, in terms of k where appropriate, of these transformations. **[5]**

(ii) Find the range of f in terms of k. **[2]**

(iii) It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case. **[6]**

June 11

7 The functions f, g and h are defined for all real values of x by

$$f(x) = |x|, \qquad g(x) = 3x + 5 \qquad \text{and} \qquad h(x) = gg(x).$$

(i) Solve the equation $g(x + 2) = f(-12)$. **[3]**

(ii) Find $h^{-1}(x)$. **[3]**

(iii) Determine the values of x for which

$$x + f(x) = 0.$$ **[2]**

Jan 2011

1 Solve the equation $|3x + 4a| = 5a$, where a is a positive constant. **[3]**

June 2010

5 **(i)** Solve the inequality $|2x + 1| \leqslant |x - 3|$. **[5]**

(ii) Given that x satisfies the inequality $|2x + 1| \leqslant |x - 3|$, find the greatest possible value of $|x + 2|$.

9 The functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - 12x \qquad \text{and} \qquad g(x) = ax + b,$$

where a and b are non-zero constants.

(i) Find the range of f. **[3]**

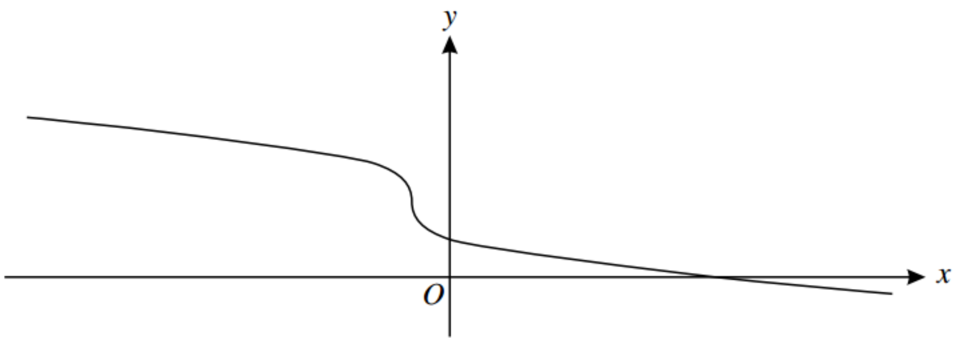
(ii) Explain why the function f has no inverse. **[2]**

(iii) Given that $g^{-1}(x) = g(x)$ for all values of x, show that $a = -1$. **[4]**

(iv) Given further that $gf(x) < 5$ for all values of x, find the set of possible values of b. **[4]**

Jan 2010

4



The function f is defined for all real values of x by

$$f(x) = 2 - \sqrt[3]{x + 1}.$$

The diagram shows the graph of $y = f(x)$.

(i) Evaluate $ff(-126)$. **[2]**

(ii) Find the set of values of x for which $f(x) = |f(x)|$. **[2]**

(iii) Find an expression for $f^{-1}(x)$. **[3]**

(iv) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. **[1]**

Question			Answer	Marks	Guidance
3	(a)		Substitute $t=3$ in $\lvert 2t-1\rvert$ and obtain value 5	B1	not awarded for final $\lvert 5\rvert$ nor for ± 5
			Substitute $t=-3$ in $\lvert 2t-1\rvert$ and apply modulus correctly to any negative value to obtain a positive value	M1	with no modulus signs remaining
			Obtain value 7 as final answer	A1	not awarded for final $\lvert 7\rvert$ nor for ± 7 NB: substitutions in $\lvert 2t+1\rvert$ will give 5 and 7 – this is 0/3, not MR; a further step to $5<t<7$ – B1 M1 A0; answers $\pm 5, \pm 7$ – this is B0 M0 A0
				[3]	
3	(b)		<u>Either</u> Attempt solution of linear equation or inequality with signs of x different	M1	
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)
			<u>Or 1</u> Attempt to square both sides	M1	obtaining at least 3 terms on each side
			Obtain $x^2-2\sqrt{2}x+2>x^2+6\sqrt{2}x+18$	A1	or equiv; or equation; condone $>$ here
			<u>Or 2</u> Attempt sketches of $y=\lvert x-\sqrt{2}\rvert$, $y=\lvert x+3\sqrt{2}\rvert$	M1	
			Obtain $x=-\sqrt{2}$ at point of intersection	A1	or equiv
			Conclude with inequality of one of the following types: $x<k\sqrt{2},\ \ x>k\sqrt{2},\ \ x<\frac{k}{\sqrt{2}},\ \ x>\frac{k}{\sqrt{2}}$	M1	any integer k
			Obtain $x<-\sqrt{2}$ or $-\sqrt{2}>x$ as final answer	A1	final answer $x<-\frac{2}{\sqrt{2}}$ (or similar unsimplified version) is A0
				[4]	
8	(i)		Attempt completion of square at least as far as $(x+2a)^2$ or differentiation to find stationary point at least as far as linear equation involving two terms	*M1	or equiv but a must be present
			Obtain $(x+2a)^2-3a^2$ or $(-2a,-3a^2)$	A1	
			Attempt inequality involving appropriate y-value	M1	dep *M; allow $<, >$ or $\le$ here; allow use of x; or unsimplified equiv
			State $y\ge -3a^2$ or $f(x)\ge -3a^2$	A1	now with $\ge$; here $x\ge -3a^2$ is A0
				[4]	

Question			Answer	Marks	Guidance
8	(ii)		Attempt composition of f and g the right way round	*M1	algebraic or (part) numerical; need to see $4x-2a$ replacing x at least once
			Obtain or imply $16x^2-3a^2$ or $144-3a^2$	A1	or less simplified equiv but with at least the brackets expanded correctly
			Attempt to find a from $fg(3)=69$	M1	dep *M
			Obtain at least $a=5$	A1	
			Attempt to solve $4x-10=x$ or $\frac{1}{4}(x+10)=x$ or $4x-10=\frac{1}{4}(x+10)$	M1	for their a; must be linear equation in one variable; condone sign slip in finding inverse of g
			Obtain $\frac{10}{3}$	A1	and no other answer
				[6]	

Question			Answer	Marks	Guidance	
1			Attempt process for finding critical values	M1	squaring both sides, 2 linear eqns, ineqs, …	If using quadratic, need to go as far as factorising or substituting in formula for M1; if using two linear eqns or ineqs, signs of $2x$ and x must be same in one, different in the other for M1
			Obtain $\frac{4}{3}$	A1		
			Obtain 6	A1		
			Attempt process for inequality involving two critical values	M1	sketch, table, …; implied by plausible soln	
			Obtain $x < \frac{4}{3}$, $x > 6$	A1	A0 for use of $\le$ and/or $\ge$	
				[5]		
7			Show composition of functions	M1	the right way round; or equiv	
			Obtain $2\sqrt[3]{12-a}+5=9$	A1	or equiv	
			Obtain $a=4$	A1		
			EITHER			
			Attempt to find $g(x)$	*M1	obtaining px^3+q or py^3+q form	
			Obtain $(2x+5)^3+4=68$	A1ft	following their value of a	
			Attempt solution of equation	M1	dep *M; earned at stage $2x+5=...$; if expanding to produce cubic equation, earned with attempt at linear and quadratic factors	
			Obtain $-\frac{1}{2}$	A1	and no others; dependent on correct work throughout	
				[7]		
			OR			
			State or imply $f(x)=g^{-1}(68)$	B2		
			Attempt solution of equation of form $2x+5=\sqrt[3]{68-4}$	M1		
			Obtain $-\frac{1}{2}$	A1		
5	(i)		State 26	B1		
			State 4	B1		
				[2]		
5	(ii)		Sketch (more or less) correct curve	B1	with approx correct curvatures and curve going through second quadrant but not fourth quadrant; allow if sketch does not meet given curve on line $y=x$	
			Refer to reflection in $y=x$ or symmetrical about $y=x$ or mirrored in $y=x$	B1	explicit reference needed, not just line $y=x$ shown on sketch	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		Attempt differentiation to find x-coordinate of stationary point or attempt completion of square as far as $(x+...)^2$	M1	or equiv; first two marks of part **(i)** may be earned by work seen in part **(ii)**; $x=-2$ only stated earns M1A1	
			Obtain $x=-2$ or $(x+2)^2$	A1	first two marks of part **(i)** are implied by correct answer to translation in x-direction	
			State translation by 2 in negative x-direction	A1	or (clear) equiv; allow correct vector	
			State translation by 4 in negative y-direction	A1	or (clear) equiv; allow correct vector	
			State stretch parallel to y-axis, scale factor k	B1	or equiv at least mentioning y and k	
				[5]		
9	(ii)		State one of $y<4k$, $y\le 4k$, $y<-4k$, $y\le -4k$ $y>4k$, $y\ge 4k$, $y>-4k$, $y\ge -4k$	B1		
			State $y\ge -4k$	B1	allow alternative notation such as $f(x)\ge -4k$ or range $\ge -4k$	
				[2]		
9	(iii)		Attempt to relate y-value involving k at their stationary point to 20 or -20 or consider discriminant of $k(x^2+4x)=20$ or of $k(x^2+4x)=-20$	*M1		
			Obtain $k=5$	A1	earned unless there is clear evidence of error in working	
			State one root $x=-2$	B1		
			Attempt solution of $k(x^2+4x)=20$	M1	dep *M; for their value of k provided positive	
			Obtain $\frac{-4\pm\sqrt{32}}{2}$	A1ft	or (unsimplified) exact equivs; following their value of k	
			Obtain $-2\pm 2\sqrt{2}$ or $-2\pm\sqrt{8}$	A1	dependent on previous A1 A1ft marks being awarded	
				[6]		

7	(i)	Either: Attempt solution of at least one linear eq'n of form $ax+b=12$	M1		
		Obtain $\frac{1}{3}$	A2	**3**	and (finally) no other answer
		Or: Attempt solution of 3-term quadratic eq'n obtained by squaring attempt at $\mathrm{g}(x+2)$ on LHS and squaring 12 or -12 on RHS	M1		
		Obtain $\frac{1}{3}$	A2	**(3)**	and (finally) no other answer
	(ii)	Either: Obtain $3(3x+5)+5$ for h	B1		
		Attempt to find inverse function	M1		of function of form $ax+b$
		Obtain $\frac{1}{9}(x-20)$	A1	**3**	or equiv in terms of x
		Or: State or imply g^{-1} is $\frac{1}{3}(x-5)$	B1		
		Attempt composition of g^{-1} with g^{-1}	M1		
		Obtain $\frac{1}{9}(x-5)-\frac{5}{3}$	A1	**(3)**	or more simplified equiv in terms of x
	(iii)	State $x \le 0$	B2	**2**	give B1 for answer $x<0$
				$\boxed{8}$	

1	Either: Obtain $\frac{1}{3}a$	B1		condone $\lvert x\rvert = \frac{1}{3}a$
	Attempt solution of linear eqn	M1		with signs of $3x$ and $5a$ different; allow M1 only if a given particular value and no recovery occurs; allow M1 only if a in terms of x attempted; allow M1 only if double inequality attempted but with no recovery to state actual values of x
	Obtain $-3a$	A1	**3**	as final answer
	Or: Obtain $9x^2+24ax+16a^2=25a^2$	B1		
	Attempt solution of 3-term quad eqn	M1		as far as substitution into correct quadratic formula or correct factorisation of their quadratic; allow M1 only if a given particular value
	Obtain $-3a$ and $\frac{1}{3}a$	A1	**(3)**	or equivs; as final answers; and no others
			$\boxed{3}$	

9	(i)	Attempt to find x-coord of staty point or complete square	M1		
		Obtain $(\frac{3}{2}, -9)$ or $4(x-\frac{3}{2})^2 - 9$ or -9	A1		or equiv
		State $f(x) \geq -9$	A1	3	using any notation; with $\geq$
	(ii)	Make one correct (perhaps general) relevant statement	B1		not 1 – 1, f is many-one, … ; maybe implied if attempt is specific to this f
		Conclude with correct evidence related to this f	B1	2	AG; (more or less) correct sketch; correct relevant calculations, …
	(iii)	Either: Attempt to find expression for g^{-1}	*M1		or equiv
		Obtain $\frac{1}{a}(x-b)$	A1		or equiv
		Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1		dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
		Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$	A1	4	AG; necessary detail required; or equiv
		[SC1: first two steps as above, then substitute $a = -1$: max possible M1A1B1]			
		[SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]			
		Or: State or imply that $y = g^{-1}(x)$ is reflection of $y = g(x)$ in line $y = x$	B1		
		State that line unchanged by this reflection is perpendicular to $y = x$	M2		
		Conclude that a is -1	A1	4	
	(iv)	State or imply that $gf(x) = -(4x^2 - 12x) + b$	B1		
		Attempt use of discriminant or relate to range of f	M1		or equiv
		Obtain $64 + 16b < 0$ or $9 + b < 5$	A1		or equiv
		Obtain $b < -4$	A1	4	
			13		

4	(i)	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (7 and hence) 0	A1	2	
	(ii)	Attempt to find x-intercept	M1		
		Obtain $x \leq 7$	A1	2	or equiv; condone use of $<$
	(iii)	Attempt correct process for finding inverse	M1		
		Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
		Obtain correct $(2-x)^3 - 1$	A1	3	or equiv in terms of x
	(iv)	Refer to reflection in $y = x$	B1	1	or clear equiv
				8	

Sep 14-13:43